Noise-robust synchronized chaotic communications

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Abstract-- Until now, research on the applications of self-synchronized chaotic circuits to communications has been hindered by the great sensitivity of self synchronized chaotic systems to additive noise. In this paper I demonstrate a self-synchronized chaotic system that synchronizes even in the presence of noise much larger than the signal. This system works because it generates signals with 2 different time scales, allowing noise added to the shorter time scale system to be averaged out by the longer time scale system. I demonstrate a simple communications scheme with this system, and I show that the curve of bit error rate as a function of (energy per bit)/(noise spectral density) is invariant with respect to bit length, allowing this system to operate in arbitrarily low signal-to-noise environments.

I. Introduction

Much of the initial research into self-synchronized chaotic systems was motivated by possible applications to communications [1-11] , for chaotic circuits are a naturally simple way to produce broad band signals. Self synchronized chaotic systems have suffered from the same problem as transmitted reference spread spectrum systems [12], however; the reference signal is contaminated by noise in the channel. It is probable that self synchronizing methods can never perform as well as stored reference methods (such as CDMA [13]) in the presence of noise, but, as I show in this work, it is possible for self synchronizing chaotic systems to be noise robust according to the definition of Abel et al. [14] , which states that a communications system is noise robust if the curve of bit error rate as a function of E_b/N_0 is invariant with respect to bit length. If a communications system is noise robust, operation in arbitrarily low signal-to-noise ratios is possible simply by increasing bit length.

II. 2-Frequency Circuit

It is possible in nature to have chaotic systems where motion occurs on very different time scales; in ferromagnetic resonance experiments in Yttrium Iron Garnet, for example [15-18], the spin magnetic moment of the material responds at frequencies in the range of 10^9 Hz and 10^3 Hz. For a previous paper, a chaotic circuit was designed which also had motion on 2 different time scales [19]. The circuit itself could be divided into a high frequency part and a low frequency part. A signal from the drive circuit was transmitted in order to synchronize a response circuit, but the signal was only coupled into the high frequency part. The low frequency part of the circuit then acted as a filter, and averaged out most of the noise added to the transmitted signal, resulting in small synchronization errors for the low frequency part even when the signal to noise ratio for the transmitted signal was less than 0 dB.

Synchronization alone is not enough, of course; it must also be possible to transmit an information signal. In this paper, the 2-frequency system is used to simulate a simple phase modulation communications scheme, and the performance of the communications scheme is measured when noise is added to the transmitted signal.

The transmitter circuit is described by

$$
\begin{aligned}
&\frac{d\theta}{dt} = \omega \\
&\frac{dx_1}{dt} = -\alpha_1\left(0.02x_1 + 0.5x_2 + x_3 + 0.5|x_4| - k\left(s_1\sin(\theta) - \beta x_1\right)\right) \\
&\frac{dx_2}{dt} = -\alpha_1\left(-x_1 + 0.02x_2 + x_6\right) \\
&\frac{dx_3}{dt} = -\alpha_1\left(-g(x_1) + x_3\right) \\
&\frac{dx_4}{dt} = -\alpha_2\left(x_1 + 0.05x_4 + 0.5x_5 + x_6\right) \\
&\frac{dx_5}{dt} = -\alpha_2\left(-x_4 + 0.11x_5\right) \\
&\frac{dx_6}{dt} = -\alpha_3\left(-g(x_4) + x_6\right) \\
&g(x) = \begin{Bmatrix} 0 & x < 3 \\ 15(x-3) & x \geq 3 \end{Bmatrix}
\end{aligned}
\tag{1}
$$

where α_1 and α_2 may be varied to alter the relative time scales of the two parts of the circuit. The signals x_1 through x_3 form the low frequency part of the circuit, while x_4 through x_6 form the high frequency part. The signal θ is the phase control signal, used to modulate the phase of the low frequency part of the oscillator in order to communicate. The frequency ω is set equal to the peak frequency of the low frequency part of the oscillator, and the signal θ is coupled into the low frequency part in order to phase synchronize the oscillator [20]. The information signal, s_1 , is set equal to ±1, while the coupling constant k = 0.2, and β = 0.37.

Figure 1(a) shows the power spectrum S of the x_1 signal measured from eq. (1) while Fig. 1(b) shows the power spectrum S of the x_4 signal measured from eq. (1) when α_1 = 0.2 and α_2 = 10.0, so that the time scales are separated by a factor of 50. The higher frequency peak in the power spectrum, most prominent in Fig. 1(b), is at 1.1 Hz. The lower frequency peak in the power spectrum, largest in Fig. 1(a), is at 0.022 Hz. Choosing different values for α_1 and α_2 changed the ratio of the two frequencies.

The signal that was transmitted from drive to response, x_t , was a linear combination of signals from the drive system. The response circuit was described by

$$x_t = \sum_{i=1}^{6} k_i x_i^d + \eta \qquad x_r = \sum_{i=1}^{6} k_i x_i^r$$

$$\begin{aligned}
\frac{dx^r{}_1}{dt} &= -\frac{1}{RC_1}\left(\gamma_1 x^r{}_1 + 0.5x^r{}_2 + x^r{}_3 + 0.5\left|x^r{}_4\right|\right) \\
\frac{dx^r{}_2}{dt} &= -\frac{1}{RC_1}\left(-x^r{}_1 + \gamma_2 x^r{}_2 + x^r{}_6\right) \\
\frac{dx^r{}_3}{dt} &= -\frac{1}{RC_1}\left(-g\left(x^r{}_1\right) + x^r{}_3\right) \\
\frac{dx^r{}_4}{dt} &= -\frac{1}{RC_2}\left(x^r{}_1 + 0.05x^r{}_4 + 0.5x^r{}_5 + x^r{}_6 + b_4\left(x_t - x_r\right)\right) \\
\frac{dx^r{}_5}{dt} &= -\frac{1}{RC_2}\left(-x^r{}_4 + 0.11x^r{}_5 + b_5\left(x_t - x_r\right)\right) \\
\frac{dx^r{}_6}{dt} &= -\frac{1}{RC_2}\left(-g\left(x^r{}_4\right) + x^r{}_6 + b_6\left(x_t - x_r\right)\right)
\end{aligned} \qquad (2)$$

where the superscript d refers to the drive circuit and r refers to the response circuit and η is a Gaussian noise signal filtered to match the bandwidth of the response system (the determination of the bandwidth is described below). Note that the error signal x_t - x_r is fed back only into the high frequency part of the circuit, x_4 to x_6 .

The parameters k_i and b_i are set to minimize the largest Lyapunov exponent for the response circuit corresponding to eq. (2) [21, 22]. The k_i 's and b_i 's are varied by a linear optimization routine in order to minimize the largest Lyapunov exponent for the response circuit. For the parameters listed in Table 1, the largest Lyapunov exponent for the response circuit was -0.22 s^{-1} when α_1 was 0.2 and α_2 was 10.0. Varying α_1 and α_2 changed the stability of the response system, but the largest Lyapunov exponent remained negative for a large range of $\alpha's$, so the same k_i 's and b_i 's were used for all simulations.

III Communication

The systems of eqs. (1-2) were simulated in order to test the communications performance of this type of circuit. The information signal s_1 in eq. (1) was set to ±1 to simulate a binary signal. The value of s_1 determined the phase of the low frequency part of eq. (1). The receiver of eq. (2) was used to measure the phase of the low frequency part of eq. (1) in order to determine the value of s_1 . The phase detector part of the receiver was

$$\begin{aligned}
&\frac{d\theta^r}{dt} = \omega \\
&\frac{du}{dt} = \frac{dx_4^r}{dt} - \frac{\alpha_1}{100.0}u \\
&v = \operatorname{sgn}[\sin(\theta^r)]u \\
&\frac{dw}{dt} = \frac{\alpha_1}{10.0}(v - 0.1w)
\end{aligned} \tag{3}$$

The frequency ω , the frequency of the local reference oscillator in the response system, was the same frequency as in eq. (1). The signal u was a high-pass filtered version of x^r_4 , high-pass filtered because the absolute value function in eq. (2) produced a DC offset in x^r_4 when the noise signal was large. The "sgn" function is the signum function (+1 for the argument > 0, -1 for the argument < 0). The variable *w* in eq. (3) is set to zero at the start of each bit interval, and *w* is measured at the end of each bit interval to determine the value of the received bit.

The necessary bandwidth for the chaotic signal is found by measuring the bit error rate. A low pass filtered noise signal is added to a low pass filtered version of the signal x_t and the resulting bit error rate is measured at the receiver as the filter breakpoint is lowered. At some given breakpoint, the bit error rate is seen to increase, so the filter breakpoint is set larger than this value. For the system in this paper, the minimum filter breakpoint was 7.5 Hz.

Figure 2 shows the probability of bit error P_b for the system of eqs. (1-3). The dark circles show P_b for a bit length of L = 653.9 s, in which case α_1 was set to 0.2 and α_2 was set to 10.0. The open circles show P_b for a bit length of L = 1307.84 s (twice as long), with $\alpha_1 = 0.1$ and $\alpha_2 = 10.0$. The energy per bit/(noise power spectral density) (E_b/N_0) is calculated for a 2-sided noise power spectrum. The two sets of data lie along the same curve, demonstrating that the curve of P_b vs. E_b/N_0 does not depend on bit length, a property not yet seen in other transmitted reference chaotic communication systems. The solid line in Fig. 2 shows the probability of bit error for binary phase shift keying (BPSK) [23] for comparison.

The noise robustness of this system may be further explored by changing the bit length L and finding the probability of bit error at a constant value of E_b/N_0. In Figure 3, E_b/N_0 is held constant at 14.3 dB while the bit length L is varied by a factor of 32, from

653.9 s to 20,924.8 s. The slow time constant α_1 is varied at the same time the bit length is varied, so α_1 varies from 0.2 to 0.2/32 = 0.00625, while α_2 is held constant at 10.0. The upper scale in Fig. 3 shows the signal to noise ratio in decibels. The probability of bit error is seen to be roughly constant when the bit length L varies by a factor of 32, demonstrating that this self-synchronizing chaotic system is noise robust for added Gaussian noise. In addition, the performance of this communications system does not degrade when the signal to noise ratio is below 0 dB.

In order to demonstrate that the transmitted signal is truly being buried in the noise, Figure 4(a) shows the power spectrum S of the transmitted signal x_t when α_1 = 0.00625 and α_2 = 10.0. Fig. 4(b) shows the same signal with added Gaussian noise so that the S/N is approximately -43 dB (corresponding to the longest bit length used in Fig. 3). The added noise covers even the peaks in the power spectrum of x_t . The high frequency peak is about 20 dB below the noise level, while the low frequency peak is at approximately the same level as the noise.

IV Conclusions

Transmitted reference communications systems should not be noise robust, since noise is added to the reference signal, and studies for chaotic transmitted reference signals confirm this supposition [14]. Self-synchronizing chaotic systems are not transmitted reference systems such as DCSK [24, 25], but are nonlinear filters. In this way they are more general versions of periodic methods such as BPSK, which uses linear filters to isolate the signal from noise. Self-synchronizing chaotic systems are not as efficient as purely periodic systems because the nonlinear filter (the chaotic response system) has a larger bandwidth, so less noise is excluded.

Phase modulation was used in this paper as a simple example to demonstrate that communication is possible with 2-frequency synchronized chaotic systems, but it is probably not the most efficient way to use these systems. A 2-frequency system might be more useful as a filter for communication involving symbolic dynamics, in which chaotic trajectories form the communications symbols [26].

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i	k_i	b_i
1	5.34	0
2	-2.58	0
3	-3.10	0
4	1.60	-0.53
5	-0.89	0.34
6	-1.61	-0.23

Table 1. Values of k_i and b_i parameters.

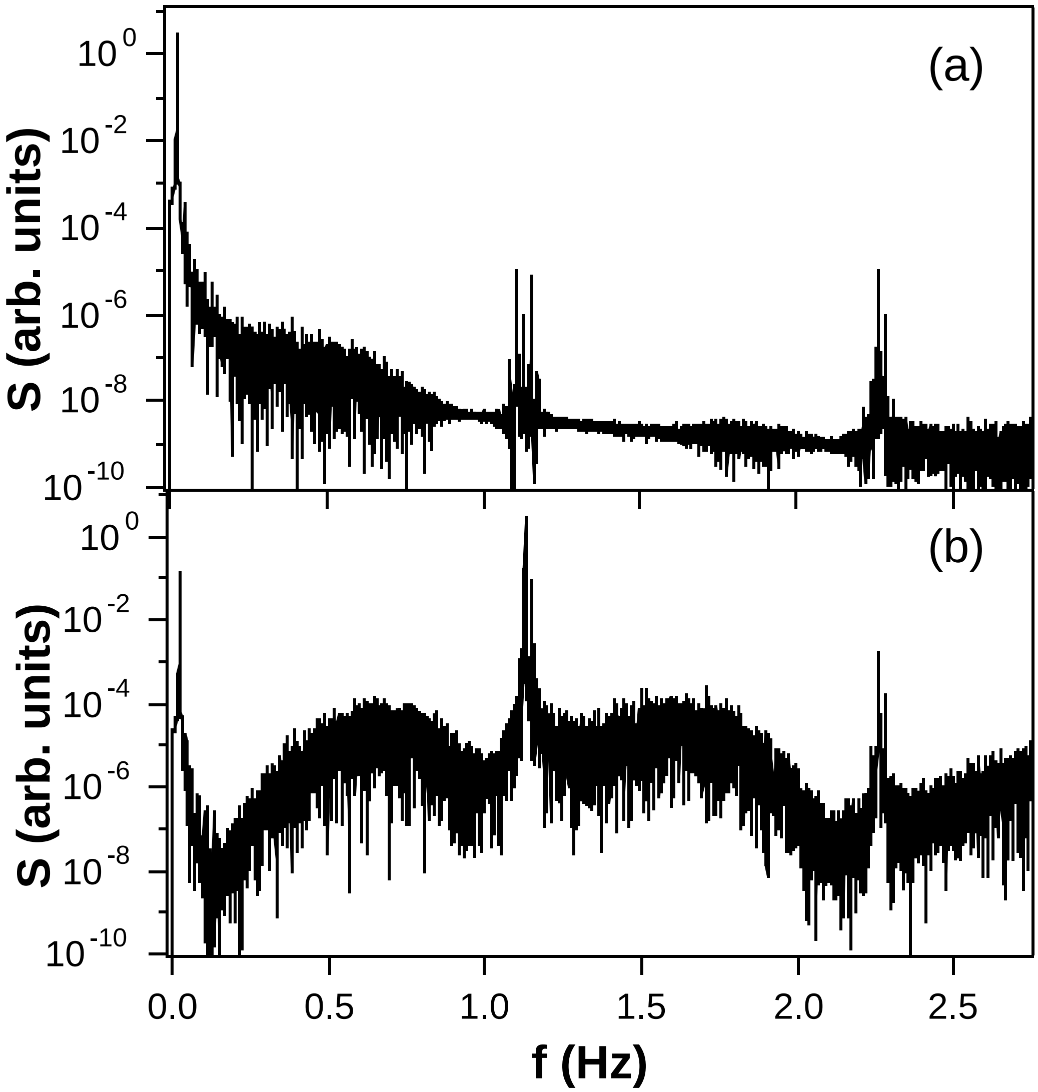

Figure 1. Power spectra S for the system of eq. (1) when $\alpha_1 = 0.2$ and $\alpha_2 = 10.0$.

(a) is the power spectrum of x_1 , while (b) is the power spectrum of x_4 .

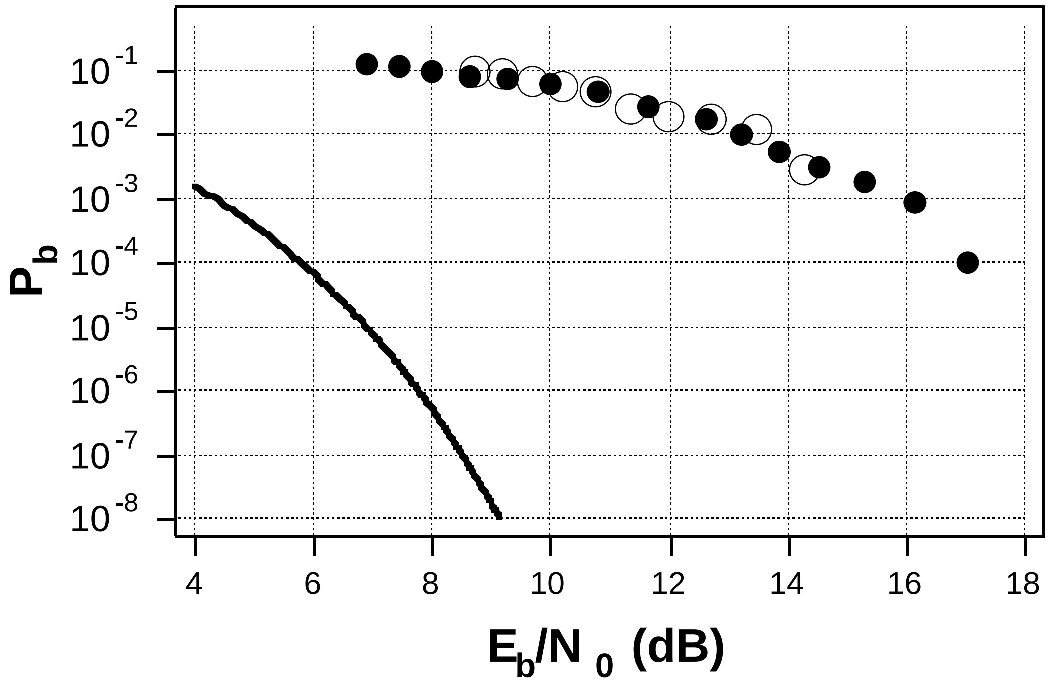

Figure 2. Probability of bit error P_b as a function of (energy per bit)/(2 sided noise power spectral density) (E_b/N_0) for the communications system described by eqs. (1-3). The filled in circles are for a bit length of L = 653.9 s, while the open circles are for L twice as long. The solid line is for binary phase shift keying (BPSK).

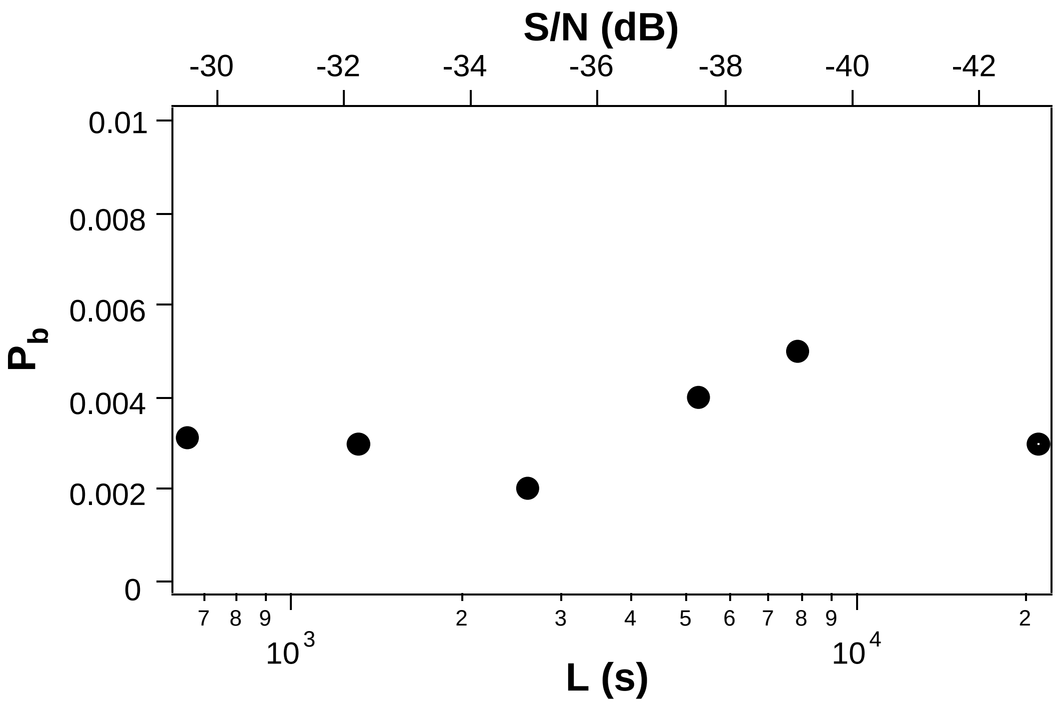

Figure 3. Probability of bit error P_b for different bit lengths L at a constant E_b/N_0 of 14.3 dB. L varies by a factor of 32. The top axis is the corresponding signal to noise ratio.

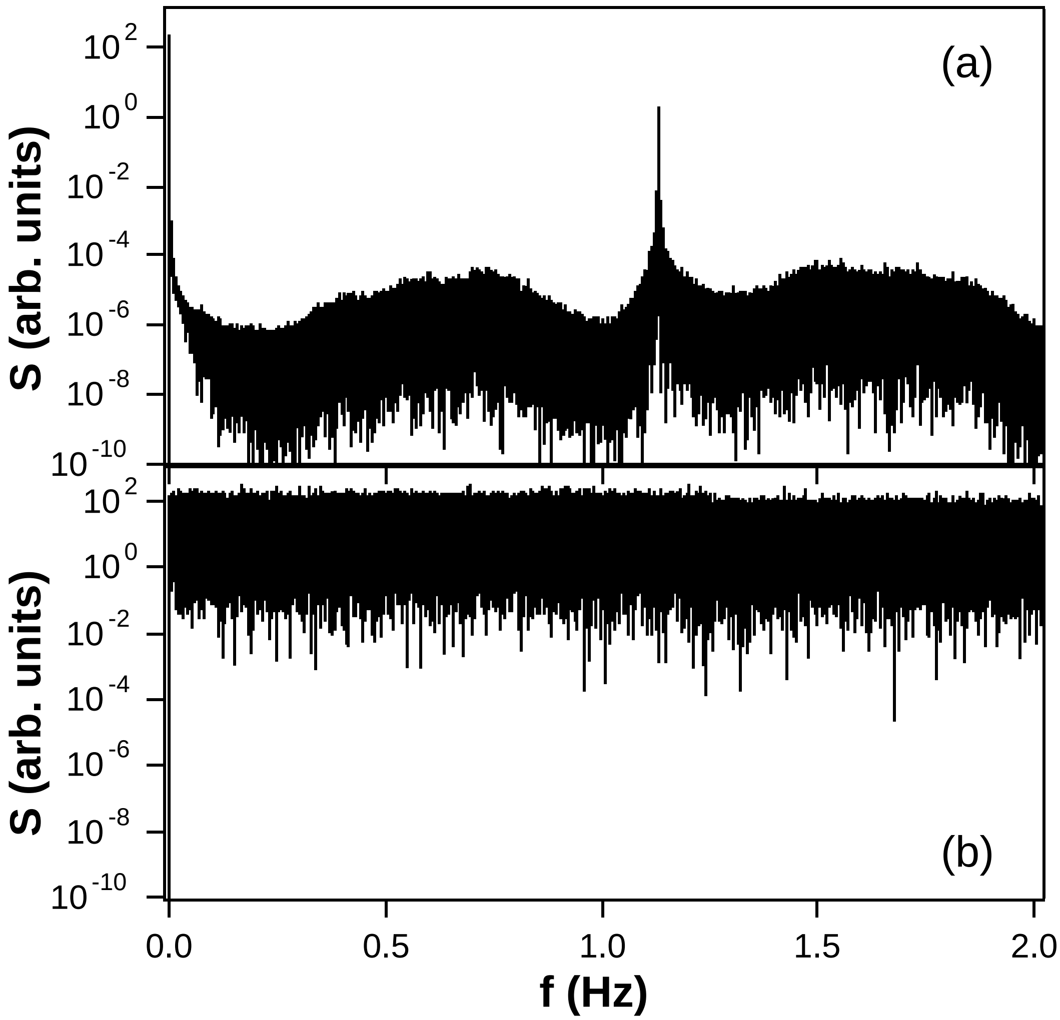

Figure 4. (a) is the power spectrum S of the transmitted signal x_t when the time factors are $\alpha_1 = 0.00625$ and $\alpha_2 = 10.0$. (b) is x_t with added noise corresponding to the largest value of L in Fig. (3), so that the signal to noise ratio is approximately -43 dB.